4 The new Keynesian model

4.1 Foundations: a classical monetary model - monetary policy

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Equilibrium: summary of the classical monetary model solution

$$n_t = \psi_{na} a_t + \nu_n \tag{1}$$

$$y_t = \psi_{ya} a_t + \nu_y \tag{2}$$

$$c_t = \psi_{ya} a_t + \nu_y \tag{3}$$

$$r_t = \rho + \sigma \psi_{ya} E_t \Delta a_{t+1} \tag{4}$$

$$w_t - p_t = \psi_{wa} a_t + \nu_w \tag{5}$$

Monetary policy

- In the classical monetary model real variables only depend on technological shocks
- This result is in contrast with the empirical literature on the effects of monetary policy on the economy
- The fact that monetary policy does not influence real outcomes does not mean that monetary policy has no role in influencing prices in the monetary model
- Another equation is needed to determine (or not) the price path. This is the monetary policy rules equation
- Note, however, that not all monetary policy rules will deliver determinacy of the price level

(In)determinacy of the price level

- In a monetary model the way monetary policy is conducted is important for determining the price level
- We will make use of the Fisher equation

$$i_t = E_t \pi_{t+1} + r_t$$

and

$$E_t \pi_{t+1} = i_t - r_t$$

note that r_t is driven only by (expected)real shocks (see equation (4)).

1. Exogenous interest rate rule

- Suppose that the central bank follows the rule

$$i_t = \rho$$

this implies

$$E_t\pi_{t+1} = \rho - r_t$$

using equation (4):

$$r_t = \rho + \sigma\psi_{ya}E_t\Delta a_{t+1} \qquad (6)$$

$$\boxed{E_t\pi_{t+1} = -\sigma\psi_{ya}E_t\Delta a_{t+1}}$$

Thus, expected inflation is determined, in the sense that it can be defined as a function of productivity shocks.

- Note, however that

$$E_t\pi_{t+1} = -\sigma\psi_{ya}E_t\Delta a_{t+1}$$

Does not pin down the price level exactly.

$$E_t\pi_{t+1} = E_t p_{t+1} - p_t$$

Which implies that any price path satisfying:

$$E_t p_{t+1} = E_t\pi_{t+1} + p_t$$

$$E_t p_{t+1} = -\sigma\psi_{ya}E_t\Delta a_{t+1} + p_t$$

Will be an also an equilibrium. For instance:

$$p_{t+1} = -\sigma\psi_{ya}E_t\Delta a_{t+1} + p_t + \xi_t$$

where ξ_t are "sunspot" shocks possibly unrelated to fundamentals (here productivity shocks) and with zero expected value $(E_t\xi_{t+1}) = 0$.

2. Inflation-based interest rate rule

Suppose that now the central bank follows a rule, reacting to current inflation

$$i_t = \rho + \phi_\pi \pi_t$$

this implies

$$E_t \pi_{t+1} = \rho + \phi_\pi \pi_t - r_t$$

Again using:

$$\hat{r}_t = r_t - \rho$$

We can write:

$$\boxed{\phi_\pi \pi_t = E_t \pi_{t+1} + \hat{r}_t}$$

2. Inflation-based interest rate rule

Let us now solve the following equation forward:

$$\pi_t = \frac{1}{\phi_\pi} E_t \pi_{t+1} + \frac{1}{\phi_\pi} \hat{r}_t \tag{7}$$

$$\pi_t = (\frac{1}{\phi_\pi})^2 E_t \pi_{t+2} + \frac{1}{\phi_\pi} \hat{r}_t + (\frac{1}{\phi_\pi})^2 E_t \hat{r}_{t+1} \tag{8}$$

...

2. Inflation-based interest rate rule

On the nth iteration we will have:

$$\pi_t = (\frac{1}{\phi_\pi})^n E_t \pi_{t+n} + \sum_{k=0}^{n} \phi_\pi^{-(k+1)} [E_t \hat{r}_{t+k}]$$

What are the stationary solutions when n goes to infinity?

- If $\phi_\pi > 1$ there is only one solution for inflation - determinacy
- If $\phi_\pi < 1$ there are many solutions - indeterminacy

2. Inflation-based interest rate rule

Suppose $\phi_\pi > 1$, then as n goes to infinity:

$$(\frac{1}{\phi_\pi})^n E_t \pi_{t+n} \to 0$$

provided expectations are stationary. Then there is only one solution to inflation and the price level:

$$\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} [E_t \hat{r}_{t+k}]$$

Inflation only depends on expectations of future real rates and these depend only on real factors.

2. Inflation-based interest rate rule

If $\phi_\pi < 1$ then:

$$(\frac{1}{\phi_\pi})^n E_t \pi_{t+n}$$

does not necessarily go to zero as n goes to infinity. Note that if we assume that:

$$E_t \pi_{t+1} = \phi_\pi (\pi_t) - \hat{r}_t$$

then the equation can be solved with stationary solutions.

2. Inflation-based interest rate rule

To see this, go back to:

$$\phi_\pi \pi_t = E_t \pi_{t+1} + \hat{r}_t$$

Substituting:

$$\phi_\pi \pi_t = \phi_\pi \pi_t - \hat{r}_t + \hat{r}_t$$

Then the equation will always be satisfied but there are many processes satisfying the condition (again, only expected inflation is determined).

2. Inflation-based interest rate rule

For example, if $\phi_\pi < 1$ then the process:

$$\pi_{t+1} = \phi_\pi \pi_t - \hat{r}_t + \xi_t$$

where ξ_t are "sunspot" shocks possibly unrelated to fundamentals (here productivity shocks) and with zero expected value, also satisfies the equilibrium. The inflation rate is not pinned down and the sunspot shocks will determine inflation

Monetary policy rules

In conclusion, in the monetary model:

- Exogenous rules lead to indeterminacy (i.e. one cannot relate inflation to the expectations of the fundamental shocks of the economy, productivity shocks)
- Feedback rules may or may not lead to indeterminacy
- In the case of a rule reacting to inflation, determinacy is only achieved if the nominal interest rate reacts more than one-to-one to inflation ($\phi_\pi > 1$) - **Taylor principle**
- The implication is that central banks should react more than proportionately to inflation to avoid that inflation does not become determined by non-fundamental shocks